Probabilistic Graphical Models
Assignment 1

Issued on: January 20, 2020

Due by: January 27, 2020 (6pm)

Guidelines for submission

Theory Problems:

- Solutions should preferably be submitted in hard copy (written solutions on A4 sheets). A submission box will be placed before deadline.
- Alternatively, a solution can be prepared in doc/latex as well. For that please export it in .pdf format (as Theory.pdf).

Programming Problems:

- You can use python/matlab for programming problems.
- Along with the main code file, please submit all required dependencies.
- Also add a report (as Code.pdf) with a brief summary of your solution.

Submit a A1_RollNo.zip file on backpack with all required files.

---

1) (4 points) Consider a standard XOR gate with inputs A, B and output C. We can easily observe using the truth table that there are two conditions when the output is 0 and two when it is 1. Now, if we consider that the output is 0, we can observe that both inputs in this case should either be 0 or 1. Consider there be a simulated XOR gate, with conditional probabilities as given in table below.

A	**B**	**P(C=1| A, B))**
0	0	0.1
0	1	0.99
1	0	0.8
1	1	0.25

Now, considering that P(A = 1) = 0.65 and P(B = 1) = 0.77 and $A \perp B$.

- (3 points) What is $P(A = 1|C = 0)$ and $P(B = 0|C = 1)$.
- (1 point) Represent the relation of A, B and C in form of a directed network.

2) (4 Points) Consider the model shown in Figure 1.

   (a) Indicate whether the following independence statements are true or false according to this model. Also Provide a very brief justification of your answer.
      i. Season $\perp$ Chills
      ii. Season $\perp$ Chills | Flu
      iii. Flu $\perp$ Dehydration
      iv. Flu $\perp$ Dehydration | Season, Nausea
      v. Chills $\perp$ Nausea
      vi. Chills $\perp$ Nausea | Headache

   (b) Does knowing you are dehydrated increase or decrease your likelihood of having the flu? Intuitively, does this make sense?

Probabilistic Graphical Models
Assignment 1

Issued on: January 20, 2020

Due by: January 27, 2020 (6pm)

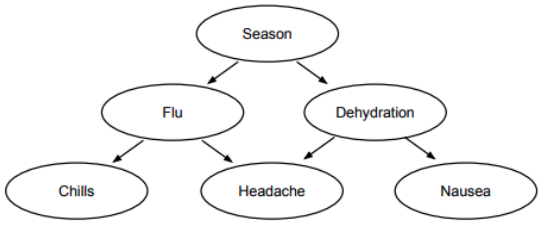

Figure 1: A Bayesian network that represents a joint distribution over the variables Season, Flu, Dehydration, Chills, Headache, and Nausea.

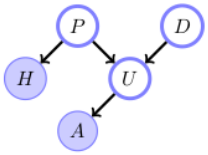

Figure 2: Here all variables are binary. P = Been to Party, H = Got a Headache, D = Demotivated at work, U = Underperform at work, A = Boss Angry. Shaded variables are observed in the true state.

3) (2 Points) Show that every node is conditionally independent of its non-descendants given its parents.

4) (2 Points) Consider the model shown in Figure 2. The boss is angry and the worker has a headache. What is the probability that the worker has been to a party? To complete the specifications, the probabilities are given as follows:

$p(U = tr \mid P = tr, D = tr) = 0.999$	$p(U = tr \mid P = fa, D = tr) = 0.9$	$p(H = tr \mid P = tr) = 0.9$
$p(U = tr \mid P = tr, D = fa) = 0.9$	$p(U = tr \mid P = fa, D = fa) = 0.01$	$p(H = tr \mid P = fa) = 0.2$
$p(A = tr \mid U = tr) = 0.95$	$p(A = tr \mid U = fa) = 0.5$	$p(P = tr) = 0.2, p(D = tr) = 0.4$

5) (4 Points) A doctor gives a patient a (D)rug (drug or no drug) dependent on their (A)ge (old or young) and (G)ender (male or female). Whether or not the patient (R)ecovers (recovers or doesn't recover) depends on all D, A, G. In addition A $\perp$ G | ϕ

   (a) Write down the belief network for the above situation.

   (b) Explain how to compute $p(recover|drug)$.

Programming Problem:

(4 points) You have studied the concept of ancestors and descendants in context of DAGs.
Write a script to find ancestors and descendants for all available nodes in a given network. Report your results on a network of minimum 5 nodes.
You can follow given steps:

- Create a function that takes input as number of nodes, connection between nodes, direction of edges etc. You can use any data structure to stores a network.
- Now write a function to find whether given trail is a path.
- Use above functions to find ancestors and descendants of each node.