Tutorial-1

1. If we know a scenario in which "If A is true then B is true". Can we also say that "if B is false then A is false"?

2. The BN of Figure 1 shows the dependencies between the four binary variables Snow, Traffic state, Risk of Falling and Lateness.
   The variable Snow (S) takes the values in $\{yes, no\}$. The value yes means there is snow on campus.
   The variable Falling (F) is in $\{yes, no\}$ depending on whether the risk of falling while travelling on foot is high or normal.
   The variable Traffic (T) takes the values in $\{normal, slow\}$. P(T = *normal*) represents the probability of a vehicle circulating on the campus to have a normal speed.
   The variable Lateness (L) takes the values in $\{yes, no\}$. The value no means that students are not late (what is normally found). The variable Lateness is yes when the number of late students exceeds a threshold.
   Note: We denote respectively *he* and *se* the values of hard evidence and soft evidence.

   Hard Evidence Case Take the case of $he_T = slow$. Now obtain,

   - $P(S = yes|he_T)$
   - $P(F = yes|he_T)$
   - $P(T = slow|he_T)$
   - $P(L = yes|he_T)$

   Soft Evidence Case Imagine that ten speed sensors are installed throughout the campus. For each sensor, the average speed recorded on the last hour is considered. We can reasonably assume that if all sensors indicate that traffic is slow down on their measuring point, then P(T = *slow*) = 1 and vice versa. If the traffic is well distributed on campus, we can also assume that the number n_{normal} of sensors that indicate a normal traffic is a good measure of the probability that the traffic is normal on campus, ie P(T = *normal*) = $n_{normal}/10$. In this context, assume that 7 speed sensors indicate that traffic is slow down, and 3 indicate a normal speed. This observation is certain. We denote this soft evidence $se_T = (0.3, 0.7)$. Now obtain,

   - $P(S = yes|se_T)$
   - $P(F = yes|se_T)$
   - $P(T = slow|se_T)$
   - $P(L = yes|se_T)$

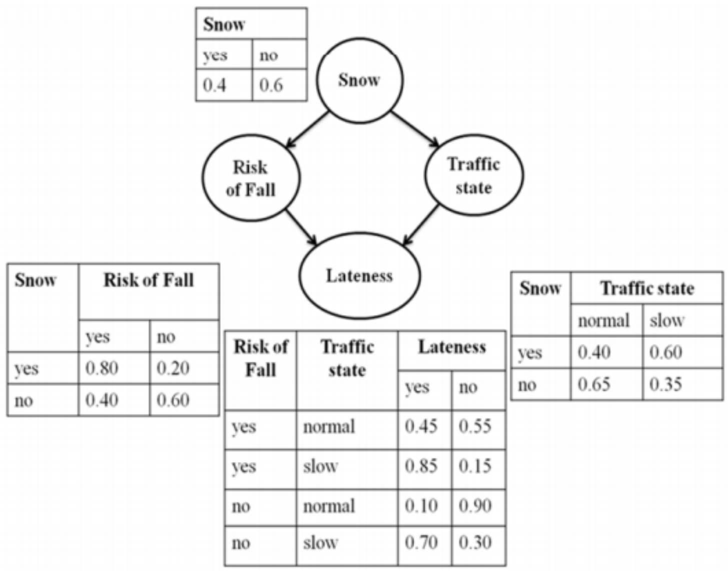
3. Consider BN in Figure 2, we need to see independence between node a and b given nodes marked in yellow. Convert this Bayesian to an undirected Markov network, applying morality wherever required, show if above independence holds.

4. We here consider Gibbs distributions where the factors only depend on two variables at a time. The probability density or mass functions over d random variables $x_1, ..., x_d$, then take the form

$$p(x_1......x_d) \propto \prod_{i \leq j} \phi_{ij}(x_i, x_j) \tag{1}$$

   These models are typically called pairwise Markov networks.

   - Let $p(x_1......x_d) \propto exp(-\frac{1}{2}x^T Ax - b^T x)$, where A is symmetric and $x = (x_1, ..., x_d)^T$. What are the corresponding factors ϕ_{ij} for $i \leq j$?
   - For $p(x_1......x_d) \propto exp(-\frac{1}{2}x^T Ax - b^T x)$, can we say that $x_i \perp x_j | \{x_1......x_d\} \setminus \{x_i, x_j\}$



Snow	
yes	no
0.4	0.6

Snow	Risk of Fall	
	yes	no
yes	0.80	0.20
no	0.40	0.60

Risk of Fall	Traffic state	Lateness	
		yes	no
yes	normal	0.45	0.55
yes	slow	0.85	0.15
no	normal	0.10	0.90
no	slow	0.70	0.30

Snow	Traffic state	
	normal	slow
yes	0.40	0.60
no	0.65	0.35

Figure 1: BN of the snow-lateness example

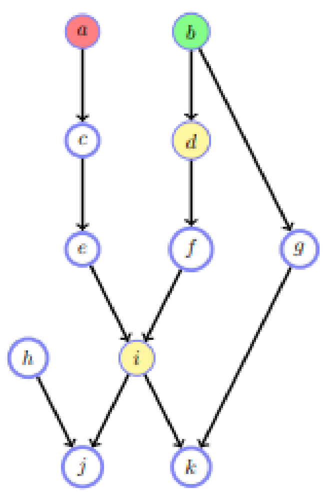


Figure 2: BN for Prob-3