

PGM Quiz-1 Solution

Ques. Sally comes home to find that the burglar alarm is sounding ($A=1$). Has she been burgled ($B=1$) or was the alarm triggered by an earthquake ($E=1$)? She turns the radio ON for earthquake and finds that radio broadcasts an earthquake alert ($R=1$).

Tables.

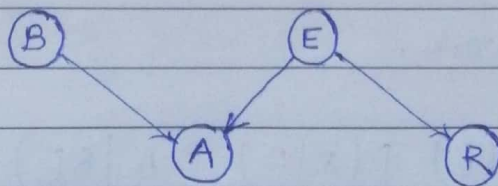
#①	Alarm = 1	Burglar	Earthquake
	0.9999	1	1
	0.99	1	0
	0.99	0	1
	0.0001	0	0

#②	Radio = 1	Earthquake
	1	1
	0	0

#③ $P(B=1) = 0.01$

#④ $P(E=1) = 0.000001$

1) Draw the graph.
(1 point)



2) Write the expression of joint probability.
(1 point)

$$P(B, E, A, R) = P(B) \cdot P(E) \cdot P(A|B, E) \cdot P(R|E)$$

3) Find independence and conditional independence.
(2 points)

Independence —

(a) $(B \perp E)$ for $B \perp E$ we have a v-structure in network.

(b) $(B \perp R)$
 $\propto (B \perp R | \phi)$ ie $(B \perp E | \phi)$

Conditional independence.

(a) $(B \perp R | E)$
also we have $(B \perp R | E, A)$

(b) $A \perp R | E$

4) (a) Find total parameters required if no simplification is there.
(1 point)

$$\Rightarrow 2^4 - 1 = 15$$

(b) Find total parameters required for the bayes network.
(1 point)

$$P(B) \cdot P(E) \cdot P(R|E) \cdot P(A|BE)$$

$$1 \quad 1 \quad 2 \quad 4 \quad = 8$$

5) Find paths, trails, ancestors & descendants.
(2 marks)

	Trail	Path		Trail	Path
(a) B to A	✓	✓	(b) E to A	✓	✓
E	✓	X	B	✓	X
R	✓	X	R	✓	✓
	Trail	Path		Trail	Path
(c) A to B	✓	X	(d) R to A	✓	X
E	✓	X	E	✓	X
R	✓	X	B	✓	X

Ancestors of
 $A \Rightarrow \{B, E\}$
 $E \Rightarrow \emptyset$
 $R \Rightarrow \{E\}$
 $B \Rightarrow \emptyset$

Descendants of
 $A \Rightarrow \emptyset$
 $E \Rightarrow \{A, R\}$
 $R \Rightarrow \emptyset$
 $B \Rightarrow \{A\}$

6) Find $P(B=1 | A=1)$

(1 mark) Given that the alarm sounds, what is prob that sally is burgled?

$$P(B=1 | A=1) = \frac{\sum_{E,R} P(B=1, E, A=1, R)}{\sum_{B,E,R} P(B, E, A=1, R)}$$

$$= \frac{\sum_{E,R} P(A=1 | B=1, E) \cdot P(B=1) \cdot P(E) \cdot P(R|E)}{\sum_{B,E,R} P(A=1 | B, E) \cdot P(B) \cdot P(E) \cdot P(R|E)}$$

$$\approx 0.99$$

⑦ What is the prob that sally is burgled (1 mark) given that alarm sounds and she comes to know from radio that earthquake come?

$$P(B=1 | A=1, R=1)$$

$$= \sum_E P(B=1, E, A=1, R=1)$$

$$\sum_{B,E} P(B, E, A=1, R=1)$$

$$= \sum_E P(B) \cdot P(E) \cdot P(A=1 | B=1, E) \cdot P(R=1 | E)$$

$$\sum_{B,E} (P(B) \cdot P(E) \cdot P(A=1 | B, E) \cdot P(R=1 | E))$$

$$\approx 0.01$$