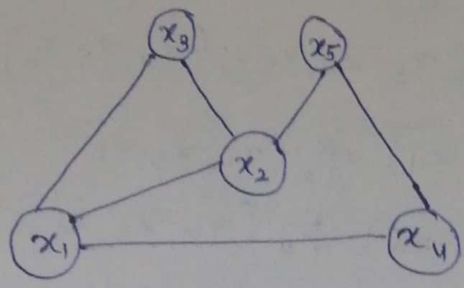
ASSIGNMENT - 02

SOLUTION

① a)



(b) Neighbours

$ne(x_1)$ — $\{x_2, x_3, x_4\}$

$ne(x_2)$ — $\{x_1, x_3, x_5\}$

$ne(x_3)$ — $\{x_1, x_2\}$

$ne(x_4)$ — $\{x_1, x_5\}$

$ne(x_5)$ — $\{x_2, x_4\}$

c) Yes, conditioning set of relation ie $\{x_1, x_2\}$ equals $ne(x_3)$, which is also the markov blanket of x_3. From this, we can infer that x_3 is conditionally independent of all other variables given $\{x_1, x_2\}$ $(x_3 \perp x_4, x_5 \mid x_1, x_2)$. So, $(x_3 \perp x_4 \mid x_1, x_2)$

d) Markov blanket

$MB(x_1) = \{x_2, x_3, x_4\}$

$MB(x_2) = \{x_1, x_3, x_5\}$

$MB(x_3) = \{x_1, x_2\}$

$MB(x_4) = \{x_1, x_5\}$

$MB(x_5) = \{x_2, x_4\}$.

Note : Markov Blanket of a node in an undirected graphical model equals the set of its neighbours.

e) First identifying all trails from x_1 to x_5.
There are three trails $x_1 - x_3 - x_2 - x_5$;
$x_1 - x_2 - x_5$; $x_1 - x_4 - x_5$.

Now, conditioning on x_2 blocks trail-1 and trail-2 and x_4 blocks trail-3 above.

So, $(x_1 \perp x_5 \mid x_2, x_4)$ $Z = \{x_2, x_4\}$

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(2)

a) There are four maximal cliques that we can figure out from given network.

(x_1, x_2, x_4) ; (x_1, x_3, x_4) ; (x_3, x_4, x_5) ; (x_4, x_5, x_6)

$$P(x_1, x_2, x_3, x_4, x_5, x_6) \propto \phi_1(x_1, x_2, x_4)\, \phi_2(x_1, x_3, x_4)\, \phi_3(x_3, x_4, x_5)\, \phi_4(x_4, x_5, x_6)$$

b) $P(x_3 | x_2, x_4) = P(x_3 | x_4)$ implies that $x_3 \perp x_2 | x_4$.

There are multiple trails connecting x_2 and x_3, one of which is $x_2 - x_1 - x_3$ which will not be blocked by conditioning on x_4.

So, we connot conclude that $P(x_3|x_2, x_4) = P(x_3|x_4)$

But, we can use global markov property to make x_2 and x_3 independent by conditioning over x_1 and x_4.

ie $P(x_3 | x_2, x_1, x_4) = P(x_3 | x_1, x_4)$

or $(x_3 \perp x_2 | x_1, x_4)$

c) $(x_1 \perp x_5 | x_2, x_3, x_4, x_6)$

The distribution that factorises according to the graph satisfies pairwise markov property here. Since x_1 and x_5 are not neighbours and x_2, x_3, x_4, x_6 are remaining nodes; the independence relation follows from the Markov property.

d) $P(x_3 | x_1, x_2, x_4, x_5, x_6) =$

$P(x_3 | x_1, x_4, x_5)$

as per local markov property.

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③ (a) $P(x_1 \ldots x_{10}) = P(x_1) \cdot P(x_2) \cdot P(x_3 | x_{10})\ P(x_4 | x_2, x_6, x_7)$
$P(x_5 | x_9) \cdot P(x_6 | x_1, x_2)\ P(x_7) \cdot P(x_8)$
$P(x_9 | x_3, x_7, x_8)\ P(x_{10} | x_2)$

Node with no parents : $\{x_1, x_2, x_7, x_8\}$

b) Pairs (i, j) for which $X_i \perp X_j$ are as :
$(1,2), (1,3), (1,5), (1,7), (1,8), (1,9), (1,10)$
$(2,7), (2,8), (3,7), (3,8), (4,8), (6,7), (6,8)$
$(7,8), (7,10), (8,10)$ — 17 such pairs.

c) $S = \{x_3, x_5, x_7, x_8, x_{10}\}$
∵ there is no active trail to above nodes from x_1 conditioning over $A = \{x_2, x_9\}$.

d) $R = \{x_1, x_5, x_6\}$.

④ a) False : Conditioning over d makes trail active.
$t \rightarrow e \rightarrow d \leftarrow b \leftarrow s$

b) True : Conditioning over s makes all trails connecting l and b inactive.

c) True : $a \perp s | l$.
even if l was not observed $a \perp s$, due to v structures at node e and node d.

d) False : Conditioning over d makes trail active.
$a \rightarrow t \rightarrow e \rightarrow d \leftarrow b \leftarrow s$

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(5) Let D : has disease T : test is +ve

$\bar{D}$: not have disease $\bar{T}$: test is -ve

$P(D) = 0.01$

$P(\bar{D}) = 0.99$

$\left.\begin{array}{l} P(T|D) = 0.99 \\ P(\bar{T}|\bar{D}) = 0.99 \end{array}\right]$ test is 99% accurate.

To compute :— $P(D|T)$

$$P(D|T) = \frac{P(T|D).P(D)}{P(T|D).P(D) + P(T|\bar{D}).P(\bar{D})}$$

$$= \frac{(0.99 \times 0.01)}{(0.99 \times 0.01) + (0.01 \times 0.99)}$$

$$= 0.5$$