Probabilistic Graphical Models
Assignment 3

Issued on: March 31, 2020 | Due by: April 7, 2020 11:59pm

Guidelines for submission

Theory Problems:

- Solutions should be submitted as a scanned Theory.pdf file of your written solutions.
- Alternatively, a solution can be prepared in doc/latex as well. For that please export it in .pdf format (as Theory.pdf).

Programming Problems:

- You can use python/matlab for programming problems.
- Along with the main code file, please submit all required dependencies.
- Also add a report (as Code.pdf) with a brief summary of your solution.

Submit a A3_RollNo.zip file on backpack with all required files.

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1. (24 points) **Factor Graphs**
   Consider a Bayesian network with distribution as :

$$P(a, b, c, d, e) = P(a|b)P(b|c, d)P(c)P(d)P(e|d) \tag{1}$$

   (a) (2 pt) Draw the network representation of the above distribution.
   (b) (2 pt) Draw a factor graph representation.
   (c) (4 pt for each part) Compute following marginals using the belief propagation:
      i. P(d,e)
      ii. P(c)
      iii. P(e)
   (d) (4 pt for each part) While computing marginal P(a,b), obtain message from factor to variable (or variable to factor) as:
      i. $\mu_{f_{bcd} \to d}(d)$ or $\mu_{d \to f_{bcd}}(d)$
      ii. $\mu_{f_{bcd} \to c}(c)$ or $\mu_{c \to f_{bcd}}(c)$
      where f_{bcd} is the factor connecting variables b,c and d in your factor graph.

2. (16 points) **Junction Tree**
   We have studied a Bayesian network as shown in Figure-1. Construct a Junction tree using this directed network showing all intermediate steps. Your triangulated graph *should contain least number of cliques or in other words cliques of largest possible size* . Take care that the resultant junction tree is indeed a "tree".
   Hint: Refer section 6.4 and 6.5 (textbook: David Barber).

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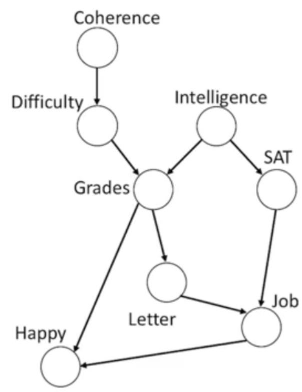

Figure 1: Network for Problem-2

3. (15 points) **Maximum Likelihood Estimation (MLE)**
X is a discrete random variable with the probability mass function in table below, where $0 \leq \theta \leq 1$ is a

X	0	1	2	3
P(X)	$2\theta/3$	$\theta/3$	$2(1-\theta)/3$	$(1-\theta)/3$

parameter. In an experiment, 10 independent observations as (1,2,0,1,2,3,1,2,0,3), were taken from the distribution.

(a) (5 pt) Obtain the likelihood function.

(b) (10 pt) Obtain the maximum likelihood estimate of θ. Using log-likelihood may simplify the problem.

4. (15 points) **Maximum A Posteriori (MAP)**
Imagine you have a friend in self-isolation due to Covid-19 lockdown and you can only communicate using binary signals. Your friend sent you a message M which can be either 0 or 1 with probabilities p and 1-p respectively. Unfortunately you received a message corrupted with a zero mean and unit variance Gaussian noise N. i.e what you received is a signal S = M + N.
Now, given that you observe S to take a value s i.e. S=s, you would like to know the value that your friend actually sent to you. Or in other words, you want to know the value m that maximizes the posterior probability $P(M = m|S = s)$. Using Bayesian rule you can obtain,

$$P(M = m|S = S) = \frac{f_{S|M}(S = s|M = m)P(M = m)}{f_S(S = s)} \tag{2}$$

(a) (5 pt) Using MAP, what will be the condition to conclude that the probable sent message was $\hat{m} = 1$.

(b) (10 pt) For what value of s can you say that the sent message was most probably $\hat{m} = 1$.

Programming Problem:
(15 Points) Write a script to implement Gaussian Naive Bayes classifier (from scratch).

- (5 pt) Implement a binary classifier for class labels 1 and 2 only.
- (5 pt) Implement a multi-class classifier with all the three classes.

Link to dataset: https://archive.ics.uci.edu/ml/datasets/seeds
Note: Add your observations in report. Report also carries 5 pts.