Probabilistic Graphical Models
Assignment 4

Issued on: April 28, 2020

Due by: May 9, 2020 11:59pm

Guidelines for submission

Theory Problems:

- Solutions should be submitted as a scanned Theory.pdf file of your written solutions.
- Alternatively, a solution can be prepared in doc/latex as well. For that please export it in .pdf format (as Theory.pdf).

Programming Problems:

- You can use python/matlab for programming problems.
- Along with the main code file, please submit all required dependencies.
- Also add a report (as Code.pdf) with a brief summary of your solution.

Submit a A4_RollNo.zip file on backpack with all required files.

---

1. (25 points) In context of importance sampling, answer the following:
   (a) (4 points) How is importance sampling better than rejection sampling?
   (b) (3 points) Considering that we cannot directly sample from the probability density function p(x), what are the properties of new function q(x) required to be used for estimation?
   (c) (8 points) When q(x) is used for sampling, how does variance differ from original Monte Carlo estimation i.e. sampling from p(x).
   (d) (3 points) Importance sampling is often called as zero variance method. In what conditions can the variance be actually reduced to zero. Why is it not practically possible to obtain zero variance.
   (e) (7 points) Consider we want to sample from an unnormalized density h(x), where $p(x) = \frac{h(x)}{Z}$ and Z is the normalization constant. How can importance sampling be used in this case for estimation?
2. (10 points) You are testing a batch of bulbs in a factory. From the produced batch, you took a set of 100 bulbs, of which 40 were found to be faulty. Let θ be the parameter, for the given Bayesian framework, obtain the posterior for θ. Take a uniform prior for θ.
3. (10 points) Explain acceptance ratio in context of Metropolis Hastings algorithm.
4. (20 points) Consider you have four rain coats, of which some are at home and others in institute. Assume that you keep travelling between the two sites. You take a rain coat with you only if it is raining, otherwise you leave it behind (i.e at home or at institute). It may also happen that all the rain coats are now at the same place implying that other place has no rain coat available. If that is the case that you were at a site with no rain coats available and it rains, you will get wet on the way.
   (a) (14 points) Considering that the probability of rain is p, what should be the probability of you getting wet?
   (b) (6 points) Assume $p = 0.6$. How many rain coats you must have to ensure that the probability of you getting wet is less than 0.1 percent.

Hint: Make a Markov chain for no of rain coats and assign probability to transitions. Then use detailed balance equation to equate transitions.

Programming Problem:
(5 Points) Write a script to approximate value of π using Monte Carlo sampling scheme.

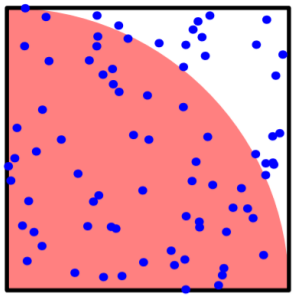

Figure 1: Sector for approximating value of π

Hint: Create a random sample generator with samples to be equally likely present in any part of quadrant.
Also, the ratio of area of quadrant and area of square is equal to $\pi/4$. Same should be the ratio for number of samples found inside quadrant to that of total number of samples in square.