Probabilistic Graphical Models
End-Sem Exam

Issued on: May 20, 2020

Due by: May 21, 2020 11:59pm

Guidelines for submission

- Solutions should be submitted as a scanned ES_RollNo.pdf file of your written solutions.
- Alternatively, a solution can be prepared in doc/latex as well. For that please export it in .pdf format (as ES_RollNo.pdf).

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1. (20 points): Consider a factor graph as shown in Figure-1 with variables $x_i \in \{0, 1\}$.

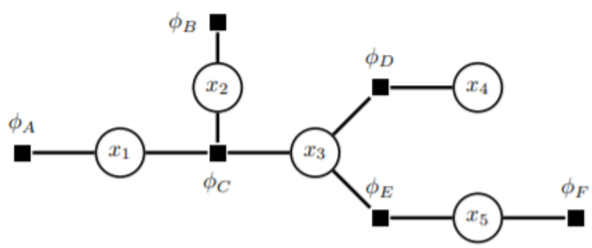

Figure 1: Factor Graph

Factors of the graph are defined as shown in the given tables:

x_1	ϕ_A
0	2
1	4

x_2	ϕ_B
0	4
1	4

x_1	x_2	x_3	ϕ_C
0	0	0	4
1	0	0	2
0	1	0	2
1	1	0	6
0	0	1	2
1	0	1	6
0	1	1	6
1	1	1	4

x_3	x_4	ϕ_D
0	0	8
1	0	2
0	1	2
1	1	6

x_3	x_5	ϕ_E
0	0	3
1	0	6
0	1	6
1	1	3

x_5	ϕ_F
0	1
1	8

Table 1: Factor definitions

As per the details shown for the factor graph shown above:

(a) (3 pts) Depict all messages required to compute $P(x_2)$. Show their respective directions using directed arrows on the factor graph.

(b) (12 pts) Using Table-1 obtain the messages required to compute $P(x_2)$.

(c) (5 pts) Compute $P(x_2 = 1)$.

2. (20 points) You have a Hidden Markov Model structure as shown in Figure-2.

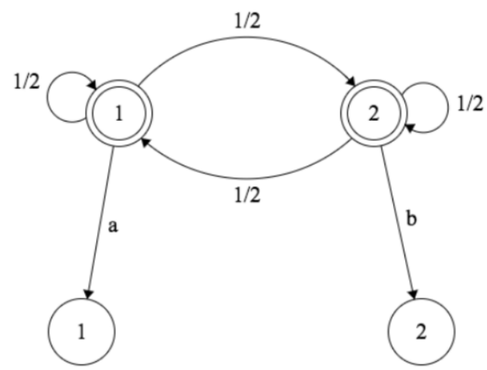

Figure 2: HMM structure

Details of the structure are as:

- Hidden states are shown using double circles.
- Emissions/outputs are shown using single circle.
- Transition probability from one hidden node to other is 1/2 as shown in Figure-2 (including self transitions).
- Emission probability or output probability follows the Poisson distribution. e.g.- when we are in state-1, it is Poisson(a). Similarly for state-2, it is Poisson(b). Here a and b are Poisson parameters (same as λ) and are constant values.
- It is equally likely to be present in state 1 and state 2.

(a) (10 pts) Obtain the probability that we are currently in state-2, given that the observed emission is 0.

(b) (10 pts) Obtain PMF of $\mathbf{X}_t$, where $\mathbf{X}_t$ is the emission at any time t.

3. (20 points) You have two identical boxes-
Box A contains 5 red and 10 green balls
Box B contains 10 red and 5 green balls
In an experiment, we will randomly select one of the boxes and sample with replacement from that box. The purpose is to determine which box we chose.
Before sampling, we assume that the "prior" probabilities are as: $P(A) = P(B) = 0.5$. Let us assume that we took n samples from the chosen box as $\mathbf{X} = (x_1, x_2, x_3, ..., x_n)$, of which k balls were observed to be red.

   (a) (7 pts) What will be the probability that you selected box-A given the observed samples?

   (b) (7 pts) What will be the probability that you selected box-B given the observed samples?

   (c) (6 pts) Let n = 100 and the k is determined by last two digits of your roll no. What is the probability that the box you chose was A given the observed samples. Here if your roll no is 19010, k shall be 10.

4. (10 points) You all have studied Chow-Liu algorithm. It initially assigns the Mutual Information (MI) between variables as the edge weights. Assume that the graph with MI edge weights is as shown in Figure-3.

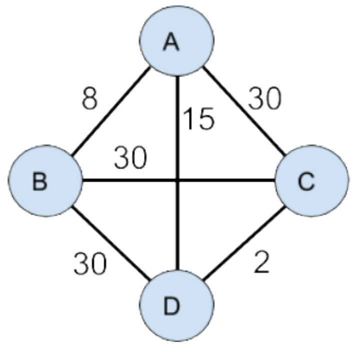

Figure 3: Graph with MI edge weights

(a) (2 pts) Find the maximum weight spanning tree.

(b) (1 pt) Transform undirected tree to directed tree

(c) (4 pts) Why do Chow Liu gives the best approximation for the tree structure?

(d) (1 pt) At most how many parents can each node have using this algorithm?

(e) (2 pts) Will the orientation of edges matter? Justify.