Quiz - 3 Solution

$$f(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{ \frac{-(x^2 - 2\rho xy + y^2)}{2(1-\rho^2)} \right\}$$

In order to use Gibbs sampling values of x and y, we need to determine the full conditional distributions for both x and y ie $f(x|y)$ and $f(y|x)$

Given that : ρ is a constant value $(\rho = 0.5)$

$$f_{Y|X}(y|X=x) = \frac{f_{XY}(x,y)}{f_X(x)}$$

- $f_X(x)$ here can be computed by marginalizing over y. which we obtain as.

$$\frac{1}{\sqrt{2\pi}} \exp\left\{ - x^2 \right\}$$

Now, $$f(y|x) = \frac{\left(\frac{\sqrt{2\pi}}{2\pi\sqrt{1-\rho^2}}\right) \exp\left\{ \frac{-(x^2 - 2\rho xy + y^2)}{2(1-\rho^2)} \right\}}{\exp\left\{ \frac{-x^2}{2} \right\}}$$

$$= \frac{1}{\sqrt{2\pi}\sqrt{1-\rho^2}} \exp\left\{ -\frac{1}{2}\left[\frac{x^2 - 2\rho xy + y^2}{1-\rho^2} - x^2 \right] \right\}$$

$$= \frac{1}{\sqrt{2\pi}\sqrt{1-\rho^2}} \exp\left\{ \frac{-1}{2}\left[\frac{y^2 - 2\rho xy + x^2\rho^2}{1-\rho^2} \right] \right\}$$

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$$= \frac{1}{\sqrt{2\pi}\sqrt{1-\rho^2}} \exp\left\{ -\frac{(y-\rho x)^2}{2(1-\rho^2)} \right\}.$$

Similarly we can obtain $f(x|y)$ as well.

we can summarize above two functions as.

$Y|X \sim N(\rho x ; 1-\rho^2) \equiv normal(\rho x, \sqrt{1-\rho^2})$

$X|Y \sim N(\rho y ; 1-\rho^2) \equiv \underbrace{normal(\rho y, \sqrt{1-\rho^2})}$

: Functions that sample from normal distributions with given μ, σ^2. (For algorithm)

Now as per Gibbs sampling criteria we will alternatively sample from one function or assign value to one variable in 1 step.

Algorithm.

Initialize :

```
x = zeros(1000, 1)        y = zeros(1000, 1)
x[1] = rnd(1)             # Assign a random value.

for ( j in 2 : 1000)
{
  # sampling from y|x
  y[j] = normal ( 0.5 * x[j-1], sqrt (1 - (0.5)^2)
```

sampling from ~~y|x~~ x|y

x[j] = normal(0.5 * y[j], sqrt(1 - 0.5*0.5))

}